

What Do Animals Think About Numbers? Continuous Magnitudes or Discretes

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Abstract

Two consistent patterns have been observed in animals regarding numerical cognition: the number size effect and the number distance effect, alluding to a mental number line that represents quantities using a logarithmic scale. Assuming the reality of this scale, this experiment tests the claim that quantity in the animal brain is represented as magnitudes rather than integers using the expectation violation paradigm. The experiment will be performed on three species, two primate and one corvid, in an attempt to verify the magnitude model, determine whether the system assumed by the model is a homology shared by the old and the new world primates and whether numerical representation in the higher aves is analogous to the primate system.

Introduction

It is a common belief that intelligence is the key factor that separates humans from their neighbors on the taxonomic tree. In accordance with this belief, it is likely that one would readily ridicule the prospect of animals performing arithmetic operations. It is common knowledge that human mathematics is founded on mankind's unique ability to observe and analyze the world through symbols. The Arabic numerals, for example, are man's extension of symbolic cognition to the quantitative elements of the world. These ideas should not suggest, however, that animals do not possess some level of understanding regarding quantitative concepts. For example, the ability to perform the operation $1+1=2$ or $3>2$, both statements that do not require verbal representation of quantities, have been observed in some animals including the primates and the Corvids used in this experiment.

Given the possibility of animal numerical cognition, one's interest may turn to the neurological functions that underlie such capabilities. From recent studies, a dichotomous model of mathematical operation in the human brain has been proposed. According to this model, human mathematics makes use of two mental arithmetic systems: a language based system dealt with primarily by the angular site of the intraparietal sulcus, and a quantity-based system dealt with by the horizontal segment of the intraparietal sulcus (Dehaene, 2004). The latter system, as referred to by the scientists who inferred its existence, is called the *number sense*. The number sense is a system where linguistic acquisition does not stand as a prerequisite to its use. Support for the existence of such a non-symbolic number system comes from a number of lesion studies. For example, "damage to the left

fusiform gyrus associated with pure alexia would create an inability to read numbers and to multiply but not to subtract or compare (multiplication requires the retrieval of verbal memory), damage to the left angular gyrus or the left horizontal intraparietal sulcus might explain the frequent double dissociations between multiplication and subtraction in patients who can still read numbers, and a lesion to the left inferior frontal gyrus might explain residual calculation abilities in patients who fail to produce the solution of arithmetic problems orally but can still solve them in writing” (Dehaene 2004).

Furthermore, “evidence suggests that (number sense) is wide spread (among the non human members of the animal kingdom)” (Dehaene, 2004). It has even been shown that there are some neurons in the brains of higher primates that fire only in response to specific quantities (Nieder, 2002). This number sense may be the founding system, an evolutionarily primitive characteristic on whose shoulder mankind’s mathematical culture stands (Gallistel, 2000).

As the psychological community begins to accept the fact that some animals may be capable of numerical cognition, two important questions are raised. The first question is what do animals think about numbers? The second is the question of whether members of phylogenetically distant taxa perceive numbers in a similar manner.

Two phenomena have proven rather useful in studies of the neural representation of numbers; the *number size effect*, where the accuracy with which two quantities are being compared decreases for two numbers equidistant from each other on the number line as the numbers being compared increases in size (often seen as the Weber’s Law applied to numbers; the ability to tell the difference between two quantities decreases proportionally with their decreasing ratio), and the *number distance effect*, where the ability to discriminate between two quantities increases proportionately with the increasing distance between them on the number line. The former will be used in this experiment. Also applied to this experiment is the expectation violation technique. In short, expectation violation studies make use of the heightened level of interest a subject shows toward events that seem to contradict the principals set forth by the subject’s own accepted logic (example: a magic show) as a tool for analyzing the subject’s thinking processes. The following is an excerpt from Exploring Animal Behavior by Paul W. Sherman and John Alcock that illustrates such an experiment. This particular study is discussed in great detail because the procedure that is proposed in testing the mental magnitude hypothesis is a derivative of this predecessor that leaves much of the basic steps of the elegant technique unchanged.

“Creative experiments – involving ‘back-stage magic’ – must be employed by scientists to determine whether rhesus monkeys can compute simple arithmetic problems. A single eggplant – a favorite food item that is sure to attract the monkey’s attention – is first

presented on “stage” in front of the animal. A screen is lowered in front of the eggplant, and a second eggplant is added behind the screen in plain view of the monkey. In control situations the experimenters then simply raise the screen, revealing two eggplants, as would be expected ($1+1=2$). In such cases the monkey will look at the two eggplants for about one second. In two test conditions, an experimenter removes an eggplant or adds a third eggplant behind the screen so that the monkey is unaware. In such instances, the unexpected outcomes ($1+1=1$) and ($1+1=3$) resulted in significantly longer “looking times” (about three to four seconds), suggesting that the monkeys detected a violation of an arithmetic operation” (pg 80).

Finally, the number size and distance effect research, taken together with fMRI and lesion studies in the area of animal numerical cognition has deduced that quantities, at least in the primate brain, are represented by a logarithmic scale (Piazza, 2004). Although primates are singled out as the most popular animals to test theories of numerical cognition (including human infants), the list of animals capable of this feat as well as some arithmetic operations now includes lions, dolphins, rodents and crows (Smirnova).

Research Proposal

Problem

According to a recent literature review, representation of quantity in the animal brain exists in the form of “mental magnitudes with scalar variability.” According to this view, numerals with higher magnitudes are associated with noisier mental signals (Gallistel, 2000). For example, it is easier, even for humans, to estimate or discriminate a quantity represented in the visual field to be either ten or fifteen, than it is to do the same for quantities fifty vs. fifty-five. The following experimental proposal attempts to test the mental magnitude hypothesis by observing whether or not the reaction of animals to the discrepancy between observed quantity and the expected quantity (expected by means of their mental arithmetic) is in accord with what would be expected under the hypothesis. If the scalar representation concept holds, it would be expected that an animal would find it more difficult to compare a large expected quantity (stored as memory in this experiment) with a large observed quantity than to compare two small quantities.

Participants

Ten Rhesus Macaques (*Macaca mulatta*), ten Cotton-top Tamarins (*Sanguinus oedipus*) and ten Hooded Crows (*Corvus cornix*) will be used in this experiment. All will be captive bred under the assumption that their brain anatomy and physiology relevant to

numerical cognition is representative of those found in the respective natural populations. The Rhesus will be used instead of a human infant because previous experiments regarding numerical cognition have been performed on this species (though the same holds for human infants) (Sherman, 2005) and since the topic of interest here is non-linguistic based numerical cognition, infants may potentially introduce language-related lurking variables. Since infants are not to be included in this experiment, the findings should not be considered applicable to them without also considering uniquely human patterns of behavior.

The purpose of including the Tamarin is to test the idea of mental magnitude representation as a primitive characteristic that arose relatively early in primate evolutionary history (Dehaene, 2004). The experiment must acknowledge both the strength and the limits of the principle of parsimony (all things being equal, the simplest explanation is the most likely). If evidence of mental magnitude usage is found in both the Rhesus and the Tamarin, parsimony would allow the conclusion that the character had evolved before the Catarrhini-Platyrrhini split (old-world/new-world monkeys). The limit of parsimony is the principle's inability to separate symplesiomorphies from parallelism. Given the similar socio-ecology shared by Tamarins and the Macaques (regarding numbers at least), the latter possibility cannot be ignored.

The purpose of the inclusion of a Corvid is to determine whether the cognitive capabilities regarding numerical cognition observed in the members of this taxon, an analogous trait to the one possessed by primates, arises from an underlying mental function similar to those of primates. A result of convergent evolution was that it endowed members of both taxa with differing neural anatomy sharing the functional similarity of representing numerals as scalar variables (this requires the assumption that Corvids, too, represent quantities on a logarithmic scale).

Materials

- ✧ 1 curtain 0.5m by 1m
- ✧ 36 wooden blocks of identical appearance measuring 0.5ft by 1ft by (1/12 ft)
- ✧ A card-board box 0.5m by 0.5m by 1m
 - Two adjacent sides of this box will be cut to form a cardboard "puppet theater" with the curtain veiled in the front to conceal how the experimenters manipulate the props (wooden blocks). A trap door is cut into the bottom of the box and will not be visible to the observer seeing the stage from the front
- ✧ A timer

Procedure

Pre-experimental training

The following pre-experimental procedures have been performed on many animals from monkeys (Brannon) to Grey parrots (Pepperberg) and therefore, will not be discussed step by step. The method by which the participants are taught to discriminate the magnitude $1/2$ from 1, however, is unique to this experiment and therefore will be discussed in sufficient detail.

Before the experiment can be performed, the animals must be trained to understand the nature of numbers associated with physical objects. Specifically, they must be taught to discriminate between the numerals from one to eight. When doing so, half of the individuals from each of the species will be taught to associate two wooden blocks glued together with the number one, two pairs as two items, three pairs as three items and so on. This group is called the scalar Ratio Group (SR). It is important to note that at this point, the animals of SR do not know that each pair of blocks can be separated. Hence, what is understood by the experimenter as two blocks (0.5ft x 1ft each) is understood by the animals as a single block (1ft x 1ft). Since in the experiment, only the front surface of the block will be visible to each animal, the width of the block can be any length so long as it is consistent for all blocks (preferably a length close to an inch). The second group, the Whole Number Ratio Group (WR) will go through the same training procedures, however they will be taught to associate a single block (0.5x1) with the quantity 1.

After the SR learns to interpret each block pair as a single object, they will then be taught the concept of a *half*. By presenting to the SR, the block pair (what is thought to be 1) and then having them observe that the single pair can be split into two equal parts that together combine to form the original quantity, the animals should grasp the principal that the magnitude of each part is $1/2$ the single unit associated with the quantity 1. This is a level of understanding that will be possible if the animals are truly representing quantities as magnitudes. If a single object is represented as a mental magnitude rather than a discrete integer, the mental noise associated with a single block (0.5 x 1) should be one half of the noise associated with a pair (1x1). The WR will not be taught the halving concept.

The Experiment

This experiment is an attempt to see whether the SR is interpreting a single block (0.5 x 1) as a discrete integer 1 or as a real-number magnitude one half of what they understand to be a single unit. A comparison will be made between the behaviors observed between the two groups within each species and of the differences observed between the WR and SR for the three species. When approaching this problem, the procedure will make use

of the expectation violation paradigm applied to this line of investigation in the past by researcher Marc D. Hauser.

The experiment will begin with the placement of an animal subject in front of the puppet theater. The curtain will be closed, but the animal will be able to see the experimenter inserting a single block of wood from above the box (0.5 x 1). The curtain will then be opened to reveal either a stack of blocks with measures 1x1 (two blocks stacked, each with a frontal surface area of 0.5 x 1) or 2x1 (four blocks stacked, each with a frontal surface area of 0.5x1). The blocks will be added through a trap door concealed to the observing animal. If the animal behaves similarly to those used in Hauser's experiment, it should be surprised by the fact that the quantity it sees behind the curtain is vastly different from what it expected to see. In other words, the observer's expectation was violated. This confusion will cause the observer to stare at the stage for a longer period after the curtain opens before losing interest, compared to those observers whose expectations would be met. The experimenter will use the stopwatch and report the time spent by the observer studying the stage after the curtain is opened. "Time spent looking" will be used as a measure of expectation violation. Comparing the results of this experiment with controls ("time spent looking" demonstrated by satisfied observers), the experimenter will note that those whose expectations were violated show a greater curiosity level.

The point of this experiment, is to understand the mental processes of the animal mind that led them to their expectations. Having established that the animals will demonstrate greater curiosity in their observation if it presents a discrepancy between the observer's expectations and their observable reality, it should now be noted that in order for an observation to violate an expectation, the two quantities between which the discrepancy occurs (the mental and the reality) must have a ratio that is large enough for the observer to notice the discrepancy. This means that the mental noises created by the quantities are small enough for the observer to discriminate one signal from the other, the signal representing the expected value in memory and the signal representing the quantity in the visual field. The number of wooden blocks that must be added behind the curtains before the observer can no longer see a discrepancy becomes a variable dependant on whether the observer perceives the initially observable block (0.5x1) as an integer 1 or a magnitude 1/2 (although the quantities are discussed here as points on a general number line, the animal brain represents them on a logarithmic scale).

The next step of this experiment requires that the initial quantity be a variable, one whose numerical distance from the violating quantity (on the general number line) is consistent. For simplicity, the distance of one "block unit = 0.5x1" will be used. Since the horizontal measure of the block is kept constant and the width is not visible, the quantitative

value of a block is determined only by its vertical measure. The following quantity pairs will be tested using expectation violation for both the SR and WR groups by presentation in a varied sequence:

Condition	Expected value	Violating value
A	0.5x1 (visible area = 0.5ft ²)	1x1 (visible area = 1ft ²)
B	1x1 (visible surface area=1ft ²)	1.5x1 (visible surface area=1.5ft ²)
C	1.5x1 (Visible surface area=1.5ft ²)	2x1 (visible surface area=2ft ²)
D	2.x1 (visible surface area=2ft ²)	2.5x1 (Visible surface area=2.5ft ²)

Since the WR group perceives each block increment as a single whole number unit, condition A will be interpreted as signal 1:signal 2, B as signal 2:signal 3, C as signal 3:signal 4 and D as signal 4:signal 5. Judging from prior studies, the WR observers will be expected to lose sight of the violation at or near condition D (otherwise, a further condition E with a 5:6 quantity ratio may be added).

The SR group, under the Mental Magnitude assumption, should be able to observe the violation of condition D because on the logarithmic scale, the ratio of 2:2.5 would be significantly less than the ratio of 4:5. Where as the WR group will be attempting to compare the noises caused by the quantity 4 with those caused by quantity 5 from the visual field, the SR will only compare the noise caused by the quantity 2 and 2.5; values whose perceived ratio is discriminable. The experimenter will continue the ratio procedure for the SR until the observers begin to show a similar level of disinterest in the stage after the curtain opens, as seen in the WR.

In exploring the significance of the difference in data obtained from the SR and the WR, a two-sample t test with 7.748047892 degrees of freedom will be used against the null hypothesis is: the average number of violating blocks at which the WR lost interest equals the average number of violating blocks at which the SR lost interest. A failure to reject the null hypothesis will lead the experimenter to either consider the possibility that the ratios used in the procedures are below the just noticeable threshold for quantity in the animal brain, consider the effect of chance variability in an experiment such as this where sample size is inevitably small, or reconsider the validity of the mental magnitude model itself. The rejection of the null hypothesis will stand as strong support for mental magnitude. Regardless of whether or not the null hypothesis is rejected, the similarities and differences in results

observed between the SR and WR for each species will be compared.

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